Problem **19**

In this problem, we essentially have a pendulum that fish are attracted to. And you observe this phenomenon. So you can take a spherical throwing we have a new fish species discovered in the Fraser River, and has been observed to be attracted to swinging pendulums as depicted in the figure, you want to observe this phenomenon. So you take a spherical metal ball of mass 0.585 kilograms attached to a rope of length 1.5 meters, and tied to a buoy as shown in the figure. If the drag force of the ball is modeled by the relationship to the velocity, times negative three, what must be the radius of this ball for oscillations to occur, we're assuming that G is 9.1 meters per second squared. And we're using a small angle approximation for pendulum so the angle of oscillation is small. And we're asked to determine when does this for what values of the radius does this oscillate. So we're going to start with a sum of moments about eight. So we're going to, I'm going to draw this in the diagram. So we have our pendulum that stuck to the buoy up here. And we're going to call this point A, and we're going to do, we're going to start with a sum of moments about a big sum of forces on there's forces pointing down due to gravity, and then there's going to be a reaction force up. But that doesn't tell us anything about the system, right? They're going to be balanced. Whereas because there's no acceleration in the vertical in the y or x directions, but we can do a sum of moments. And that's going to be equal to i alpha. So let's draw a freebody diagram of this to the side. So we have our pendulum with our sphere over here, a radius r. And that's attached at the center like that. So we're going to call this point A, and we're going to draw in all of the forces. So we have a force downwards, due to gravity. So we're going to call this F G. And this is just going to be equal to the mass times g. Right. And then we're going to have a drag force. And it depends which way you're moving. So in this diagram, I'm going to draw it in one direction, but it actually depends in which direction you're moving. So you can see that if this was moving this way, then the drag force would oppose and counteract. Instead, if this was swinging down this way, then the drag force would actually point in the opposite direction. Because of this relationship over here, there's a negative sign, which means that it counteracts the direction of the velocity, which is true for for drag forces. So you have to take this with with a grain of salt. So what I'm going to do is I'm just going to draw it in this direction. And so this is going to be f d, which is negative three times the velocity. And the velocity again, is drawn in this diagram, but it's, it's the velocity of this fear. And, again, we're going to have reaction forces over here, our y and r x. And these will cancel out because we're gonna take a sum of moments about this point. So let's do this sum of moments. So the sum of moments about a is equal to i about a times alpha. And we start with IA, so ay ay is going to be, I have this sphere at the bottom here, plus parallel axis to move at a distance L away, right, this distance over here. So I about the center, or we hear about, oh, the center is going to be equal to two fifths times m r squared, right, where r is the radius. And then we add parallel axis, so plus m, Big L squared, right? So this is going to be our i About eight. Now we can actually take the sum of moments So we're going to assume that this is our coordinate system, x, y and a positive rotation is counterclockwise. So we will take this our moments with the following convention. So we have that negative three v times the length minus m g times the length times sine theta is equal to i, which is two fifths, r squared plus l squared times alpha. Alright, so this is just I just plugged it in times alpha, this term over here, is equal to the sum of moments, which is the component due to the force from the drag times the length, because that gives us this moment here, force force, as distance perpendicular either perpendicular gives us this term. And then this term is one due to gravity. And remember, we have a sine theta, because there's an angle, right? There's an angle here. So this is defined as theta. And then there's an angle. There's this point straight down. So there's an angle that relates these two quantities, this being theta, this being 90 minus theta. But we can make the small angle approximation. So we can say that sine theta is equal to theta. And we can also say that the velocity v is equal to L times theta dot, which is also omega. And then alpha is equal to theta double dot. Right? So we replace this into the equation, and we get that negative three L squared theta dot minus m g, l times theta, because sine theta becomes theta is equal to two fifths r squared plus m L squared times theta double dot. Right. So we can condense this equation into to overfit to fits M r squared plus l squared, all times theta double dot plus three L squared theta dots plus mg L, theta is equal to zero. So you can see that this is a differential equation in theta. And we can actually solve this differential equation, but we don't need to solve it because this question is not asking. Theta is for theta with respect to time, it's only asking us to find what values of the radius make this system oscillate. So what do we need for this system to oscillate? Well, we need the characters characteristics part of the solution to be imaginary, right? Once that is imaginary, and actually oscillates, and doesn't just decay and stop, right. So, what we need is, we need to ensure that we satisfy this condition that the characteristic solution so needs, characteristics solution to be imaginary. So that we oscillate. And how do we do this? Well, we enforce the following condition, three L squared minus four times two, two fifths M r squared plus m L squared m g l has to be smaller or equal to zero, right? Because this comes directly from the solution, the way we solve this differential equation. And we want to make sure that the roots are imaginary. So the part in the square root is actually not is negative, right? So that we get a complex number. It's imaginary. And so we want this part, which is the part in the square root to be smaller or equal to zero. And this is going to set our radius. So we know all the other parameters, we know L, we know m, we know g, so we're just solving for that radius there. So isolating for R, we get that r squared has to be bigger or equal to three L to the four minus four L squared, m squared, sorry, l squared G. Sorry, this is L cubed divided by eight fifths times m squared, G, Big L. And if we plug in values and we take the square root, we get that R must be bigger or equal to 0.05 meters and this is the final answer.